Version 1.0

**General Certificate of Education (A-level)
January 2012**

**Mathematics**                                                    **MFP4**

**(Specification 6360)**

**Further Pure 4**

# Final

# Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

## Key to mark scheme abbreviations

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| $-x$ EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MFP4**

| Q | Solution | Marks | Total | Comments |
|---|----------|-------|-------|----------|
| **1** | Use of $ab\cos\theta = \mathbf{a}\cdot\mathbf{b} = 21$ $\Rightarrow \cos\theta = \dfrac{7}{5\sqrt{2}}$ | M1 A1 | | |
| | $\Rightarrow \sin\theta = \dfrac{1}{5\sqrt{2}}$ | B1 ft | | FT exact only |
| | Use of $\lvert\mathbf{a}\times\mathbf{b}\rvert = ab\sin\theta = 3$ | M1 A1 | 5 | CSO |
| | **Total** | | **5** | |
| **2(a)** | Reflection      in $x = z$ | M1 A1 | 2 | |
| **(b)** | Rotation      about the $y$-axis | M1 A1 | | |
| | Through $\cos^{-1}0.6\ (\approx 53.13^{\circ})$ | A1 | 3 | Ignore direction |
| | **Total** | | **5** | |
| **3(a)** | Char. Eqn. is $\lambda^2 - 8\lambda - 9 = 0$ | M1 | | Attempted |
| | Quadratic solved to get two roots | dM1 | | |
| | $\Rightarrow \lambda = 9,\ -1$ | A1 | | |
| | Subst$^{\mathrm{g}}$. back $\lambda$ (at least once) : | M1 | | |
| | $\lambda = 9 \Rightarrow -x + y = 0$ $\Rightarrow \lambda = 9$ has evecs. $\alpha\begin{bmatrix}1\\1\end{bmatrix}$ | A1 | | any $\alpha \neq 0$ |
| | $\lambda = -1 \Rightarrow x + y = 0$ $\Rightarrow \lambda = -1$ has evecs. $\beta\begin{bmatrix}1\\-1\end{bmatrix}$ | A1 | 6 | any $\beta \neq 0$ |
| **(b)** | $(0, 0)$ | B1 | 1 | |
| | **Total** | | **7** | |

| Q | Solution | Marks | Total | Comments |
|---|----------|-------|-------|----------|
| **4(a)** | $\mathbf{X\,X^T} = \begin{bmatrix} 3 & x \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ x & 7 \end{bmatrix}$ | M1 | | Attempted multn. with $\mathbf{X^T}$ correct |
| | $= \begin{bmatrix} x^2+9 & 7x-3 \\ 7x-3 & 50 \end{bmatrix}$ | A1 | 2 | |
| **(b)** | $\mathbf{X^T X} = \begin{bmatrix} 3 & -1 \\ x & 7 \end{bmatrix}\begin{bmatrix} 3 & x \\ -1 & 7 \end{bmatrix}$ | | | |
| | $= \begin{bmatrix} 10 & 3x-7 \\ 3x-7 & x^2+49 \end{bmatrix}$ | M1 | | Good attempt |
| | $\mathbf{X\,X^T} - \mathbf{X^T X} = \begin{bmatrix} x^2-1 & 4x+4 \\ 4x+4 & 1-x^2 \end{bmatrix}$ | M1 | | Good attempt |
| | $\text{Det}(\mathbf{X\,X^T} - \mathbf{X^T X}) = \begin{vmatrix} x^2-1 & 4x+4 \\ 4x+4 & 1-x^2 \end{vmatrix}$ | M1 | | Good attempt to factorise/expand the determinant |
| | $= (x+1)^2 \begin{vmatrix} x-1 & 4 \\ 4 & 1-x \end{vmatrix}$ | | | |
| | $= -(x+1)^2\left\{(x-1)^2+16\right\} \le 0$ for all real $x$ | E1 | 4 | Explained/demonstrated fully |
| **(c)** | $x = -1$ | B1 | 1 | CSO |
| | **Total** | | **7** | |

| Q | Solution | Marks | Total | Comments |
|---|----------|-------|-------|----------|
| **5(a)** | $\begin{vmatrix} 2 & n & 1 \\ 3 & -1 & n \\ -1 & 7 & 1 \end{vmatrix}$ <br><br> Expanding the det. of the coefft. mtx. <br> Setting it = 0 <br> Obtaining & solving a quadratic eqn. in $n$ <br><br> $0 = n^2 + 17n - 18 = (n + 18)(n - 1)$ <br> $\Rightarrow n = 1, -18$ | M1 <br> M1 <br> M1 <br><br><br> A1 | <br><br><br><br><br><br> 4 | <br><br><br><br><br><br> CSO |
| **(b)** | $n = 1$ gives $\quad 2x + y + z = 5$ <br> $\qquad\qquad\qquad 3x - y + z = 1$ <br> $\qquad\qquad\qquad -x + 7y + z = 1$ <br><br> Eliminating one variable from a pair of equations, twice <br><br> e.g. ② – ① $\Rightarrow x - 2y = -4$ <br> $\qquad$ and ② – ③ $\Rightarrow 4x - 8y = 0$ <br><br> Inconsistency clearly demonstrated from fully correct working <br><br> 3 planes have no common intersection (or form a $\Delta^r$ prism) | B1 <br><br><br><br> M1 <br><br> A1 ft <br> A1 ft <br><br> E1 <br><br><br> B1 ft | <br><br><br><br><br><br><br><br><br><br><br><br> 6 | ft their chosen integer $n$ <br><br><br><br><br><br><br><br><br><br> Also ft "3 planes meet in a common line" or "3 planes form a sheaf" if consistency conclusion made |
| | **Total** | | **10** | |

| Q | Solution | Marks | Total | Comments |
|---|----------|-------|-------|----------|
| **6(a)** | Use of scalar product on $\begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}$ | M1 | | |
| | Sc.Prod. $= \pm 21$ | B1 | | |
| | Moduli $\sqrt{54}$ and $\sqrt{26}$ correct | B1 | | Accept 7.348… & 5.099… |
| | AWRT $56^{\circ}$ | A1 | 4 | From correct working |
| **(b)** | $2x + y + 7t = 10$ and $3x + y - 4t = 7$ noted or used | M1 | | |
| | Eliminating (say) $y$ to get $x$ as a fn. of $t$ | M1 | | |
| | $x = 11t - 3$ | A1 | | CAO |
| | Subst$^{\text{g}}$. back for $y$ | M1 | | |
| | $y = 16 - 29t$ | A1 | | CAO |
| | $\dfrac{x+3}{11} = \dfrac{y-16}{-29} = z \ \ (=t)$ | B1 ft | 6 | |
| **(c)** | Attempt at either $\begin{pmatrix} \lambda + 20 \\ 9\lambda - 1 \\ 4\lambda + 7 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix} = 10$ or $\begin{pmatrix} \lambda + 20 \\ 9\lambda - 1 \\ 4\lambda + 7 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} = 7$ | M1 | | |
| | Solving either $40 + 2\lambda - 1 + 9\lambda + 49 + 28\lambda = 10$ or $60 + 3\lambda - 1 + 9\lambda - 28 - 16\lambda = 7$ | M1 | | |
| | $\lambda_1 = -2 \qquad \qquad \lambda_2 = 6$ | A1A1 | | |
| | $P = (18, -19, -1)$ and $Q = (26, 53, 31)$ | | | |
| | $PQ = \sqrt{8^2 + 72^2 + 32^2} = \sqrt{6272} = 56\sqrt{2}$ | M1A1 | 6 | **NB** $P$, $Q$ not required: $d = \mid \lambda_1 - \lambda_2 \mid \times \mid \mathbf{i} + 9\mathbf{j} + 4\mathbf{k} \mid$ $= 8 \times 7\sqrt{2} = 56\sqrt{2}$ **M1 A1** |
| | **Total** | | **16** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)** | $\begin{bmatrix} c & s \\ -s & c \end{bmatrix}$ | B1 | | |
| | $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$ | M1 | | |
| | $= \begin{bmatrix} cX + sY \\ -sX + cY \end{bmatrix}$ | A1 | 3 | |
| **(b)(i)** | $(cX + sY)^2 - 6(cX + sY)(-sX + cY) - 7(-sX + cY)^2 = 8$ | | | |
| | $(c^2X^2 + 2csXY + s^2Y^2) - 6([c^2 - s^2]XY + sc[Y^2 - X^2])$ $-7(s^2X^2 - 2csXY + c^2Y^2) = 8$ | M1 | | Substn. for $x$ & $y$ in eqn. *and* multiplying out |
| | $p = c^2 + 6sc - 7s^2$ | A1 | | |
| | $q = 16cs - 6(c^2 - s^2)$ | A1 | | AG |
| | $r = s^2 - 6sc - 7c^2$ | A1 | 4 | |
| **(ii)** | Factorising: $3s^2 + 8sc - 3c^2 = (3s - c)(s + 3c) = 0$ | M1A1 | | Or by double angles |
| | Deducing a tan value | M1 | | |
| | $\tan\theta = \frac{1}{3}$  ($\theta$ acute) | A1 | | |
| | $\cos\theta = \dfrac{3}{\sqrt{10}}$ ,  $\sin\theta = \dfrac{1}{\sqrt{10}}$ | A1 | | Both |
| | Subst$^g$. sensible values back for $p$ and $r$ | M1 | | |
| | $2X^2 - 8Y^2 = 8$ | A1 | | |
| | $\dfrac{X^2}{2^2} - \dfrac{Y^2}{1^2} = 1$ | A1 | 8 | CSO |
| **(iii)** | Since $C'$ is a hyperbola, and it is just $C$ rotated, it follows that $C$ is a hyperbola | E1 | 1 | |
| | **Total** | | **16** | |

**MFP4 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **8** | For considering $\begin{vmatrix} 1 & 2n & n-1 \\ n & 2n^2+n & n^2-1 \\ n^2 & -1 & 1-n^2 \end{vmatrix}$ | B1 | | Or by scalar triple product |
| | $= (n-1)\begin{vmatrix} 1 & 2n & 1 \\ n & 2n^2+n & n+1 \\ n^2 & -1 & -1-n \end{vmatrix}$ | M1A1 | | For 1$^{st}$ factor |
| | $= (n-1)\begin{vmatrix} 1 & 2n & 1 \\ n & 2n^2+n & n+1 \\ n(n+1) & (n+1)(2n-1) & 0 \end{vmatrix}$ | M1 | | Row ops. for 2$^{nd}$ factor |
| | $R_3{}' = R_3 + R_2$ | | | |
| | $= (n-1)(n+1)\begin{vmatrix} 1 & 2n & 1 \\ n & 2n^2+n & n+1 \\ n & 2n-1 & 0 \end{vmatrix}$ | M1A1 | | |
| | $= (n-1)(n+1)$ $\left\{ 2n^3 + 2n^2 + 2n^2 - n - 2n^3 - n^2 - 2n^2 - n + 1 \right\}$ | | | |
| | **OR** | | | |
| | $= (n-1)(n+1)\begin{vmatrix} 1 & 2n & 1 \\ 0 & n & 1 \\ n & 2n-1 & 0 \end{vmatrix}$ | | | |
| | $R_2{}' = R_2 - nR_1 =$ $(n-1)(n+1)\left\{ 2n^2 - n^2 - 2n + 1 \right\}$ | M1 | | Full method for remaining factors |
| | $= (n-1)(n+1)(n-1)^2$ | A1 | | |
| | $n = -1$ | B1 | 9 | CSO |
| | | | | **Note**: Expanding straightaway scores **B1 M1** and then **A1** for $n^4 - 2n^3 + 2n - 1$ Thereafter, **M1 A1** for 1$^{st}$ factor, **M1** for 2$^{nd}$ factor attempted and **M1** for full method for remaining factors plus **A1** and **B1 cso** at the end, as above. |
| | **Total** | | **9** | |
| | **TOTAL** | | **75** | |